On Exponential Intuitionistic Fuzzy Number

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Abstract:
The methods for finding critical path using exponential fuzzy numbers are inadequate. In this paper, we have introduced the new concept of exponential intuitionistic fuzzy numbers. Also, a method of ranking exponential intuitionistic fuzzy number is proposed and a numerical example is illustrated to find the critical path using the proposed method.

Key words:
Exponential intuitionistic fuzzy number, Ranking, Critical Path

1. Introduction

In many fields like decision making, control theory etc., vagueness of numerical quantities may occur frequently, which led to the introduced of fuzzy sets by Zadeh [10] in 1965. The concept of fuzzy numbers has been further developed by many authors. Later on Atanassov generalized the concept of fuzzy set and introduced the idea of Intuitionistic fuzzy set [1]. The concept of fuzzy numbers and arithmetic operations with these numbers were first introduced and investigated by Chang and Zadeh, Atanassov and Garagov [2] proposed the notion of the interval-valued intuitionistic fuzzy set (IVIFS), which is characterized by a membership function and a non-membership function whose values are intervals rather than exact numbers. IVIFNs have huge amount of application in decision making processes. Further developments of IFS theory, including intuitionistic fuzzy geometry, intuitionistic fuzzy topology, intuitionistic fuzzy logic, an intuitionistic fuzzy approach to artificial intelligence. Since they are very useful and powerful tool in modeling imprecision or uncertainty, valuable applications of IFSs have been developed in many different fields, including pattern
On Fuzzy Upper and Lower Almost $\beta$-continuous Multifunctions

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Abstract:
The aim of this paper is to introduce and study fuzzy upper and fuzzy lower almost $\beta$-continuous and weakly $\beta$-continuous multifunctions. Also, several characterizations and properties of these multifunctions along with their mutual relationships are established in $L$-fuzzy topological spaces. Later, composition and union between these multifunctions are investigated.

Key words and phrases: $L$-fuzzy topology, fuzzy multifunction, fuzzy upper and fuzzy lower almost $\beta$-continuous, weakly $\beta$-continuous, composition and union.

1. Introduction and preliminaries

Kubiak [19] and Sostak [28] introduced the notion of ($L$-) fuzzy topological space as a generalization of $L$-topological spaces (originally called ($L$-) fuzzy topological spaces by Chang [10] and Goguen [12]). It is the grade of openness of an $L$-fuzzy set. A general approach to the study of topological types structures on fuzzy powersets was developed in [13-15, 19, 20, 28-30].

Berge [9] introduced the concept multiampling $F : X \to Y$ where $X$ and $Y$ are topological spaces and Popa [25, 26] introduced the notion of irresolute multiampling. After Chang introduced the concept of fuzzy topology [10], continuity of multifunctions in fuzzy topological spaces have been defined and studied by many authors from different view points (e.g. see [5, 6, 22-24]). Tisporkova et al., [31, 32] introduced the Continuity of fuzzy multivalued mappings in the Chang’s fuzzy topology [10]. Later, Abbas et al., [2] introduced the concepts of fuzzy upper and fuzzy lower semi-continuous multifunctions on $L$-fuzzy topological spaces in Sostak sense.

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Abstract:
The aim of this paper is to introduce the notion of not quasi-coincidence $(\overline{q})$ of a fuzzy point and the concept of $(\overline{e}, \overline{e} \lor \overline{q}_k)$-fuzzy ideals in $d$-algebras. The criteria for a fuzzy ideal to be an $(\overline{e}, \overline{e} \lor \overline{q}_k)$-fuzzy ideal is established, $(\overline{e}, \overline{e} \lor \overline{q}_k)$-fuzzy ideal and their Cartesian product and union are discussed. Some properties of $(\overline{e}, \overline{e} \lor \overline{q}_k)$-fuzzy ideals under homomorphism are also investigated.

Keywords:
$d$-algebra, Fuzzy $d$-ideal, Doubt fuzzy ideal, $(\overline{e}, \overline{e} \lor q)$-fuzzy ideal, $(\overline{e}, \overline{e} \lor \overline{q}_k)$-fuzzy ideal.

1. Introduction

In 1991 Xi [13] applied the concept of fuzzy sets to $BCK$-algebras which were introduced by Imai and Iseki [6] in 1996. Neggers and Kim [12] introduced the class of $d$-algebras which is a generalization of $BCK$-algebras and investigated relation between $d$-algebras and $BCK$-algebras. Akram and Dar [1] introduced the concepts fuzzy $d$-algebra, they introduced fuzzy subalgebra and fuzzy $d$-ideals of $d$-algebras. Bhakat and das [5] used the relation of ‘belongs to’ and ‘quasi-coincident’ between fuzzy point and fuzzy set to introduced the concept of $(\overline{e}, \overline{e} \lor q)$-fuzzy subgroup and $(\overline{e}, \overline{e} \lor q)$-fuzzy subring. Basnet and Singh [4] introduced $(\overline{e}, \overline{e} \lor q)$-fuzzy ideals of $BG$-algebra in 2011. It is now natural to investigate similar type of generalization of the existing fuzzy subsystem with other algebraic structure. Reza Ameri et al. [2] introduced the notion of $(\overline{e}, \overline{e} \land q_k)$-fuzzy subalgebras in $BCK/BCI$-algebras. In this paper, we
Soft Rough Approach to Lattice-ideal

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Abstract:
Rough and soft sets are two different mathematical tools for dealing with uncertainties. Soft rough set is the study on roughness through soft set. Soft rough set is a fusion, proposed by Feng et al. [6] between the two mathematical approaches to vagueness. The aim of this paper is to study the lattice theory in the framework of soft rough set. We consider the soft approximation space by means of soft set and define the notions of upper and lower soft rough ideals in a lattice. A numerical example is presented to support of our proposed study.

Keywords:

1. Introduction
In 1999, Molodtsov [4] proposed soft set theory as a new mathematical tool for dealing with uncertainties. The operations of soft set are defined by Maji et al. [13] and redefined by Çağman and Enginoğlu [12]. Recently, the properties and applications on the soft set have been studied increasingly. For example, see [10], [11], [19].

Rough set was initiated by Pawlak [20] as a formal tool for modeling and processing incomplete information in information system. Rough set consists of two key notions: rough set approximations and information systems. Rough set approximations are defined by means of an equivalence relation namely indiscernibility relation. Every rough set is associated with two crisp sets, called lower and upper approximations and viewed as the sets of elements which certainly and possibly belong to the set. It has been studied successfully to knowledge discovery, decision analysis, inductive reasoning, mereology and many other fields.

Many interesting and meaningful applications in the field of mathematics, computer science and other related fields have been designed with the help of Pawlak’s rough set.
Application of Generalized Weakly Compatibility in Common Fixed Point Results on Fuzzy Metric Spaces

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Abstract:
We introduce the concepts of generalized compatibility and generalized weakly compatibility for the pair \( \{ F, G \} \) of mappings \( F, G : X \times X \to X \) in the setting of fuzzy metric space and also introduced the concept of common fixed point of the mappings \( F, G : X \times X \to X \). We establish a common fixed point theorem for generalized weakly compatible pair \( F, G : X \times X \to X \), without mixed monotone property of any of the mappings, on a non-complete fuzzy metric space, which is not partially ordered. We also given an example to validate our result. Our results generalize some recent comparable results in the literature.

Key words and phrases:
Coupled coincidence point, coupled fixed point, common fixed point, fuzzy metric space, generalized compatible mappings, generalized weakly compatible mappings.

1. Introduction

Many authors have been studied the concept of fuzzy metric spaces in fuzzy topology under different points of view. In particular, George and Veeramani [2] introduced and studied the notion of fuzzy metric with the help of continuous \( t \)-norms, which modified the concept of fuzzy metric space given by Kramosil and Michalek [8] and from now on, we refer this type of fuzzy metric. Many authors studied the existence of common or coupled fixed points in the setting of fuzzy metric spaces including [6, 7, 10, 12, 13, 14]. Bhaskar and Lakshmikantham [1] introduced the notion of coupled fixed point, mixed monotone mappings and gave some coupled fixed point theorem and applied these to study the existence and uniqueness of solution for periodic boundary value problems. Lakshmikan-tham and Ciric [9] proved coupled coincidence and common coupled fixed point theorems for nonlinear contractive mappings in partially ordered complete metric spaces and extended the results of Bhaskar and Lakshmikantham [1]. Sedghi et al. [12] gave a coupled fixed point theorem for contractions in fuzzy metric space, which was...
On Preserving Intuitionistic Fuzzy \( g \)-closed Sets

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Abstract:
In this paper we extend the concepts of \( a \)-closed and \( a \)-continuous mappings due to Baker [5] in intuitionistic fuzzy topological spaces and obtain several results concerning the preservation of intuitionistic fuzzy \( g \)-closed sets. Further more we characterize intuitionistic fuzzy \( T_{1/2} \)-spaces due to Thakur and Chaturvedi [16] in terms of intuitionistic fuzzy \( a \)-continuous and intuitionistic fuzzy \( a \)-closed mappings and obtain some of the basic properties and characterization of these mappings.

Key words:
Intuitionistic fuzzy \( g \)-closed sets, Intuitionistic fuzzy \( g \)-open sets, Intuitionistic fuzzy \( g \)-continuous, Intuitionistic fuzzy \( a \)-closed, Intuitionistic fuzzy \( a \)-continuous and Intuitionistic fuzzy \( gc \)-irresolute mappings.

1. Introduction

After the introduction of fuzzy sets by Zadeh [19] in 1965 and fuzzy topology by Chang [6] in 1968, several researches were conducted on the generalizations of the notion of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [7] introduced the concept of intuitionistic fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [9], fuzzy connectedness [15], fuzzy separation axioms [4, 11, 13], fuzzy metric spaces [14], fuzzy continuity [10], fuzzy \( g \)-closed set [16], fuzzy \( g \)-continuity [18] and fuzzy \( gc \)-irresolute mappings [17] have been generalized for intuitionistic fuzzy topological spaces. In this paper we shall introduce the concepts of intuitionistic fuzzy \( a \)-closed and intuitionistic fuzzy \( a \)-continuous mappings using intuitionistic fuzzy \( g \)-closed sets. This definition enables us to obtain conditions under which maps and inverse maps preserve intuitionistic fuzzy \( g \)-closed sets. We also characterize intuitionistic fuzzy \( T_{1/2} \)-spaces in terms of intuitionistic fuzzy \( a \)-continuous and intuitionistic fuzzy \( a \)-closed mappings. Finally some of basic
Intuitionistic Fuzzy Graphs: Weakening and Strengthening Members of A Group

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Abstract:
Ross and Harary used graph theory to characterize the role of a members of a group with respect to the member’s presence causing the graph of the group to be more highly connected than that when the member was absent, while a weakening members is one whose presence causes the graph to be more weakly connected. Takeda and Nishida presented an application of a fuzzy graph to the group structure in order to be able to consider situations where the members and edges between members have different strength of membership in the graph. In this paper, we consider intuitionistic fuzzy graphs.

Keywords:
Intuitionistic fuzzy graphs; weakening and strengthening members; directed edge; directed path; involutive fuzzy complement; transitive closure.

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Multi-fuzzy Vector Space and Multi-fuzzy Linear Transformation over A Finite Dimensional Multi-fuzzy Set

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Abstract:
The main goal of this article is to introduce and study the multi-fuzzy vector spaces and its properties. Here we present an internal characterization of multi-fuzzy linear transformation and derived fuzzy rank-nullity theorem. The direct sum for multi-fuzzy vector spaces under certain conditions is also characterized. Some properties and characterization for projection operators are established.

Keywords:
Multi-fuzzy vector spaces, Multi-fuzzy linear transformation, Direct sum, Projection operator.

1. Introduction

The theory of fuzzy sets which was introduced by Zadeh [20] is applied to many mathematical branches. Goguen generalized them to the notion of $L$-fuzzy set [6]. Since then several special $L$-fuzzy set such as the interval-valued fuzzy set, the intuitionistic fuzzy set, the three dimensional fuzzy set, the interval-valued intuitionistic fuzzy set and the type-2 fuzzy set have been proposed. Great progress, both in theory and in application, has been made based on these $L$-fuzzy sets. On the other hand, few years after the inception of the notion of fuzzy set, Rosenfeld [16] started the pioneer work in the domain of fuzzification of the algebraic objects, with his work on fuzzy group.

Algebraic structures play a prominent role in the mathematics with wide range of applications in many disciplines such as theoretical physics, computer science, coding theory, topological spaces, etc. This provides sufficient motivation to the researchers to review various concepts and results from the area of abstract algebra in the broader framework of fuzzy setting. One of the structures which is most extensively discussed in the mathematics and its applications is lattice theory.
Non-commuting Mappings and Common Fixed Points in Fuzzy Metric Spaces

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Abstract:
In this paper, we state and prove two common fixed point theorems for non-commuting maps in fuzzy metric spaces in the sense of Kramosil and Michalek, using the notions of compatibility, reciprocal continuity and E. A. property. Our work contains extensions of the fixed point theorems mainly due to R. P. Pant in classical metric spaces and perhaps the first fixed point theorem in fuzzy metric spaces guaranteeing the existence of a common fixed point of a family of maps when all the maps may be discontinuous and even many may not satisfy the compatibility conditions. We deduce some corollaries to our theorems and also illustrate them with suitable examples.

Keywords:
Fuzzy metric spaces, compatible maps, reciprocal continuous maps, E. A. property, common fixed points.

1. Introduction

In search of obtaining an appropriate and consistent notion of fuzzy metric spaces, Deng [6], Erceg [7], Kaleva and Seikkala [17], Kramosil and Michalek [18] introduced the notion in various ways. In particular, Kramosil and Michalek introduced the notion in the year 1975 by generalizing the concept of probabilistic metric spaces introduced by Menger to fuzzy setting. Later in the year 1994, George and Veeramani [8] modified the notion of fuzzy metric spaces, introduced by Kramosil and Michalek with the help of continuous $t$-norms and obtained a Hausdorff topology for this kind of fuzzy metric spaces. They further showed that every metric $d$ on $X$ induces a fuzzy metric $M_d$ (the...
Generalized Weakly Compatible Pair of Mappings and Its Application in Common Fixed Point Results on Modified Intuitionistic Fuzzy Metric Spaces

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Abstract:
We introduce the concept of generalized compatibility and generalized weakly compatibility for the pair \( \{F,G\} \), of mappings \( F,G : X \times X \to X \) in the setting of modified intuitionistic fuzzy metric space and also introduce the concept of common fixed point of the mappings \( F,G : X \times X \to X \). We establish a common fixed point theorem for generalized weakly compatible pair \( F,G : X \times X \to X \), without mixed monotone property of any of the mappings, on a non complete modified intuitionistic fuzzy metric space, which is not partially ordered. We also give an example to validate our result. Our results generalize some recent comparable results in the literature.

Key words and phrases:
Coupled coincidence point, coupled fixed point, common fixed point, modified intuitionistic fuzzy metric space, generalized compatible mappings, generalized weakly compatible mappings.

1. Introduction

Kramosil and Michalek [15] generalized the concept of probabilistic metric space to the fuzzy framework, which was later modified by George and Veeramani [10] with the help of continuous \( t \)-norms and defined the Hausdorff and first countable topology on fuzzy metric spaces. This topology can also be constructed on each fuzzy metric space in the sense of Kramosil and Michalek [15] and it is metrizable.

Atanassov [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets, which motivated many researcher to work around it. Alaca et al. [1] using the idea of intuitionistic fuzzy sets defined the notion of intuitionistic fuzzy metric space with the help of continuous \( t \)-norm and continuous \( t \)-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [15]. In [17] Park...
Cubic Structure of $BG$-subalgebras of $BG$-algebras

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Abstract:
In this paper, the notion of cubic $BG$-subalgebras of $BG$-algebras are introduced. The homomorphic image and inverse image of cubic $BG$-subalgebras are studied and investigated some related properties.

Keywords and phrases:
$BG$-algebra, $BG$-subalgebra, cubic set, cubic $BG$-subalgebra.

1. Introduction

Extending the concept of fuzzy sets ($FSs$), many scholars introduced various notions of higher-order $FSs$. Among them, interval-valued fuzzy sets ($IVFSs$) provides with a flexible mathematical framework to cope with imperfect and imprecise information. Moreover, Jun et al. [6] introduced the concept of cubic sets, as a generalization of fuzzy set and interval-valued fuzzy set. Jun et al. [7] applied the notion of cubic sets to a group, and introduced the notion of cubic subgroups.

A Study on Fuzzy Soft $\tilde{g}F_\sigma$ Sets

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Abstract:
In this paper, the concept of fuzzy soft $\tilde{g}F_\sigma$ sets are introduced and its interrelations with other types of fuzzy soft closed sets are studied with suitable counter examples. Equivalently the interrelations of fuzzy soft $\tilde{g}F_\sigma$ continuous functions with other types of fuzzy soft continuous functions are discussed with necessary counter examples.

Keywords
Fuzzy soft $\tilde{g}F_\sigma$ sets, Fuzzy soft $\tilde{g}F_\sigma$ continuous functions.

1. Introduction

In many complicated problems arising in the fields of engineering, social science, economics, medical science etc., involving uncertainties, classical methods are found to be inadequate in recent times Molodstove [9] introduced soft set theory as different method for vagueness in 1999. He applied soft set theory, Riemann integration, perron integration, smoothness of functions. Maji et al [7] defined new notions of soft set theory Shabir and Naz [13] defined soft topology by using soft sets and presented the basic properties in their paper. Fuzzy soft set which is a combination of fuzzy sets and soft sets were first introduced by Majii.et.al [8] in 2001. Many researchers improved this study and gave new results ([2], [3]). Tanaya and Kandemis [14] defined fuzzy soft topology on a fuzzy soft set over an initial universe. The concept of $\tilde{g}$-open set was discussed by Rajesh and Erdal Ekici [11]. The concept of $\tilde{g}$-closed set was discussed by B. Amudhambigai, M. K. Uma and E. Roja [1]. Fuzzy $G_\delta$ set was introduced by Balasubramanian [5]. In this paper the concept of fuzzy soft $\tilde{g}F_\sigma$ sets are introduced and its interrelations with other types of fuzzy soft closed sets are studied with suitable counter examples. Equivalently the interrelations of fuzzy soft $\tilde{g}F_\sigma$ continuous functions with other types of fuzzy soft continuous functions are discussed with necessary counter examples.
Some Observations on Completeness and Compactness in Fuzzy Normed Linear Spaces

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Abstract:
In this paper, concept of \( l \)-fuzzy convergence sequence, \( l \)-fuzzy closed, \( l \)-fuzzy complete and \( l \)-fuzzy compact sets are introduced in fuzzy normed liner spaces (with general \( t \)-norm setting) and some observations are made in our earlier paper [3]. Based on these concepts, we have studied completeness and compactness of finite dimensional fuzzy normed linear spaces and extended the celebrated Riesz Lemma.

Key words:
\( l \)-fuzzy convergent, \( l \)-fuzzy closed, \( l \)-fuzzy complete, \( l \)-fuzzy compact, fuzzy normed linear space.

0. Introduction

Katsaras [6], first introduced in 1984, the idea of fuzzy norm on a linear space. After that Felbin, Cheng & Mordeson introduced the idea of fuzzy norm on a linear space in different approach (for reference please see [4, 5]). Following Cheng & Mordeson [4], we have introduced in [1], a definition of fuzzy norm whose associated fuzzy metric is similar to that of Kramosil & Michalek [8]. In this definition we took “min” as \( t \)-norm in the triangle inequality (N4) and established a decomposition theorem which played a crucial role in developing fuzzy functional analysis (for reference please see [2, 9, 10]). We are now considering the issue of generalizing the restriction on the underlying \( t \)-norm as it is very vital in the fuzzification process. To do this, in our earlier paper [3], we have considered fuzzy normed linear spaces with the underlying \( t \)-norm in its general form and establish some important results involving completeness and compactness in finite dimensional fuzzy normed linear spaces including Riesz Lemma.

Recently it is observed that the notion of closed sets which has been introduced in our earlier paper [3] is not applicable to establish Riesz Lemma. To rectify this error, in this paper, we introduce the concept of \( l \)-fuzzy convergence, \( l \)-fuzzy closed set, \( l \)-fuzzy
On The Solution of Linear Time-varying Differential Dynamical Systems with Fuzzy Initial Condition and Fuzzy Inputs

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Abstract:
In this paper we investigate the solutions of linear time-varying differential dynamical systems with fuzzy initial condition and fuzzy inputs. We use a complex number representation of the \( \alpha \)-level sets of the fuzzy states to characterize the solutions of such systems by a closed form formula which could be easily used in practical computations. Examples are given to illustrate the results.

Keywords:
Fuzzy number; Fuzzy differential equation; Fuzzy initial condition.

1. Introduction

Let \( \mathbb{R} \), \( \mathbb{R}^+ \) denote the set of all real numbers and non-negative real numbers, respectively. \( \mathcal{F} \) denotes the set of all fuzzy numbers on \( \mathbb{R} \). \( \mathbb{C} \) denotes the set of all complex numbers. \( C\left([t_0, t_1]; \mathbb{R}^{n \times m}\right) \) denotes the continuous real matrix \( (n \times m) \) valued functions defined on time-interval \([t_0, t_1]\).

In this paper we investigate the solutions of the fuzzy differential dynamical systems of the type:

\[
\begin{align*}
\dot{x}(t) &= A(t)x(t) + B(t)u(t) \\
x(t_0) &= X_0
\end{align*}
\] (1)
γ-connectedness in L-topological Spaces

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Abstract:
In this paper, first we study the concept of γ-closed sets in L-topological spaces. Then after a new kind of connectivity called γ-connectedness in L-topological spaces is introduced by help of γ-closed sets in L-topological spaces. Some fundamental properties of γ-connectedness are studied and the inter-relationship between P-connectedness, S-connectedness and connectedness are explained. It is also included that the famous K. Fan’s Theorem can be extended to L-topological spaces for γ-connectedness.

Keywords:
γ-open set, γ-closed set, γ-separated set, γ-connectedness, L-topological space.

1. Introduction and preliminaries

In general L-topological spaces, where L is a fuzzy lattice, Wang introduced connectivity in [13]. Connectedness [3] is a well-known notion in topology. Numerous authors presented different kinds of connectivities in L-topological spaces in the Chang’s [6] sense. Here we first study the concept of γ-open set in L-topological spaces. Our aim is to introduced the concept of γ-connectedness in L-topological spaces with the notion of γ-open set in L-topological spaces. Further we study some of the fundamental properties of γ-connectedness. The famous Theorem of K. Fan’s can be generalized to L-topological spaces for γ-connectedness. Finally we establish the inter-relationship between P-connectedness, S-connectedness and connectedness.

Throughout this paper, X and Y denote non-empty ordinary sets and $L = L(\leq, \lor, \land)$ always denotes a fuzzy lattice and 0 and 1 are the smallest and the greatest element of L respectively. By a fuzzy lattice we mean a complete completely distributive lattice L, if L has an order reversing involution $a \rightarrow a'$ ($a \in L$). A mapping from X into L is said to be an L-fuzzy set on X. The collection of all L-
Uniform Integrability of Set-valued Random Variables on Capacity Spaces

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Abstract:
The article aims at discussing the uniform integrability of the sequence of set-valued random variables on capacity spaces. We investigate the space of the Choquet integrably bounded set-valued random variables, and in this space, we introduce the concepts of uniform integrability.

Keywords:
Set-Valued Random Variable; uniformly integrable; Capacity space.

1. Introduction

It is well known that the concept of capacity [7] has been usually used to deal with some uncertain phenomena which can not be easily modeled by using probability measure. Capacity is non-additive measure and the Choquet integral is one kind of nonlinear expectations. Many papers developed the Choquet theory and its applications (e.g. [1], [6], [9], [10], [11], [17], [19], [21]). There is also another way to deal with uncertain phenomena, i.e. set-valued random variables (also called random sets, multifunctions, correspondences in literature) and the Aumann integral [3]. The theory of set-valued random variables and set-valued stochastic processes with applications was developed very deeply and extensively in the past 40 years (e.g. [2,4,5,12,13,14,15,16,22,23]). However, there are many complex systems in which we have to deal with above two uncertainty phenomena at the same time. So it is necessary to investigate further the connections between the theory of set-valued random variables and the Choquet theory. Wang and Li [20] proved Fatou’s Lemmas, Lesbesgue dominated convergence theorem and monotone convergence theorem of sequences of the set-valued Choquet integrals, i.e. the integrals of set-valued random variables with respect to capacities.

In this paper, we shall focus on the uniform integrability of the sequence of set-valued random variables on capacity spaces. We shall extend the concepts of uniform
Some Results on $G$-fuzzy Product Topological Spaces

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Abstract:
$G$-fuzzy topological spaces was introduced by Mathews and Samuel [2008] in their papers titled $G$-fuzzy topological spaces and fuzzy compactness using two operations sum $\Theta$ and conjunction $\&$. In this paper, we give a brief introduction to $g$-Fuzzy Product topological spaces. We make an attempt to motivate $g$-fuzzy product topological properties.

Keywords:

1. Basic concept

We give a brief account of the developments right from fuzzy sets and fuzzy topological spaces up to compactness and product space in $g$-fuzzy topological spaces. Let us start with basic concepts of $g$ fuzzy topological spaces.

Definition 1.1. Let $X$ be a non empty set. A fuzzy set $A$ in $X$ is characterized by a membership function $\mu : X \rightarrow [0,1]$ where $[0,1]$ is the closed unit interval, while an ordinary set $A \subseteq X$ is identified with its characteristic function $\mu_A : X \rightarrow \{0,1\}$.

We use the same symbols, capital letters $A, B, C \cdots$ to denote both fuzzy sets and sets in classical set theory. Membership functions of fuzzy sets $A, B, C \cdots$ are denoted by $\mu_A, \mu_B, \mu_C \cdots$. If $A$ denotes a fuzzy set, $\mu_A(x)$ is called the grade of membership of $x$ in $A$. 
L-fuzzy Ideals with Thresholds-\((\alpha, \beta]\) of Lattices

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Abstract:
The aim of this paper is to extend the concept of a fuzzy sublattice (ideal) by introducing L-fuzzy sublattice (ideal) with thresholds \((\alpha, \beta]\) of a bounded lattice. Some properties of L-fuzzy sublattice (ideal) with thresholds \((\alpha, \beta]\) of a bounded lattice are investigated. L-fuzzy convex sublattice with thresholds \((\alpha, \beta]\) of a bounded lattice is studied. Finally, we establish a one-one correspondence between L-fuzzy ideals with thresholds \((\alpha, \beta]\) of a bounded lattice and their isomorphic images.

Keywords:
L-fuzzy sublattice with thresholds \((\alpha, \beta]\), L-fuzzy convex sublattice with thresholds \((\alpha, \beta]\), L-fuzzy ideal with thresholds \((\alpha, \beta]\), and L-fuzzy prime ideal with thresholds \((\alpha, \beta]\).

1. Introduction

After the introduction of fuzzy sets by Zadeh [16], there have been a number of generalizations of this fundamental concept. A new type of fuzzy subgroup, \((\alpha, \beta]\)-fuzzy subgroup where \(\alpha, \beta\) are any two of \(\{\epsilon, q, \epsilon \lor q, \epsilon \land q\}\) with \(\alpha \neq \epsilon \land q\), was introduced in an earlier paper of Bhakat and Das [2] by using the combined notions of “belongingness” and “quasicoincidence” of fuzzy points and fuzzy sets, which was introduced by Pu and Liu [11]. In fact, the \((\epsilon, \epsilon \lor q)\)-fuzzy subgroup is an important generalization of Rosenfeld’s fuzzy subgroup [12]. Jun [7] introduced the concept of
An Introduction of Product Topology in $g$-fuzzy Topological Spaces

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Abstract:
In this paper, we introduce product topology in $g$-Fuzzy Topological Spaces. We make an attempt to discuss some basic properties.

Key words:
Fuzzy set, $g$-fuzzy topology, Base for fuzzy topological space, $g$-fuzzy Subspace Topology, $g$-fuzzy Compactness, $g$-fuzzy connectedness and $g$-fuzzy product topology.

1. Introduction

Since the concept of fuzzy set was introduced by Zadeh [12] in a paper published in 1965, a number of research work have been dedicated on development of various aspects of the theory and applications of fuzzy sets. The fuzzy topological space was introduced by Chang [3] and since then various notions in classical topology have been extended to fuzzy topological spaces. Mathews and Samuel [8] suggested an alternate and more general definition of fuzzy topological spaces called $g$-fuzzy topological space.

2. Basic concepts

We give a brief account of the developments right from fuzzy sets and fuzzy topological spaces up to $g$-compactness and product space in $g$-fuzzy topological spaces.